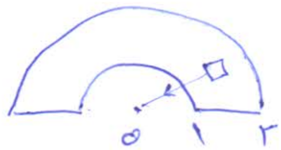
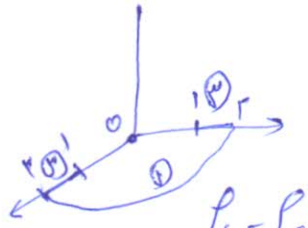
حل تکلیف سری اول الکترومغناطیس



$ds = R\,dR\,d\varphi$

$\bar{r} = -R\hat{a}_R \rightarrow E = \iint \frac{\rho_0 \cos\varphi \, R\,dR\,d\varphi\,(-R\hat{a}_R)}{4\pi\varepsilon_0 R^3}$

الف) $\rightarrow \bar{E} = \frac{\rho_0}{4\pi\varepsilon_0}\int_{\varphi=0}^{\pi}\cos\varphi\,d\varphi \int_{R=1}^{2}\frac{dR}{R}(-\hat{a}_R) = \frac{\rho_0 \ln 2}{4\pi\varepsilon_0}\hat{a}_R$



$E = \int_{\varphi=0}^{\pi/2}\frac{\rho_0\,2\,d\varphi\,(-2\hat{a}_R)}{4\pi\varepsilon_0 \times 8} = \frac{\rho_0}{8\pi\varepsilon_0}\times\frac{\pi}{2}(-\hat{a}_R)$

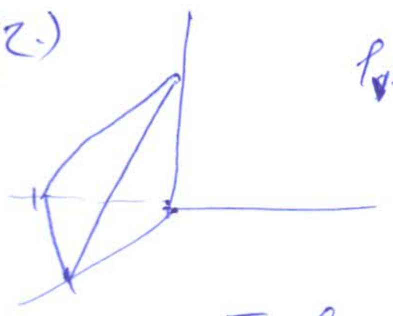
$\rho_L = \rho_0 \qquad dl = 2\,d\varphi$

ب)

$E_2 = E_3 = \int_{R=1}^{2}\frac{\rho_0\,dR\,(-R\hat{a}_R)}{4\pi\varepsilon_0 R^3} = \frac{\rho_0}{4\pi\varepsilon_0}\int_{R=1}^{2}-\frac{dR}{R^2}\hat{a}_R = +\frac{\rho_0}{8\pi\varepsilon_0}(-\hat{a}_R)$

$\bar{E} = E_1 + E_2 + E_3 = \left(\frac{\rho_0}{16\varepsilon_0} + \frac{\rho_0}{4\pi\varepsilon_0}\right)(-\hat{a}_R)$

ج)



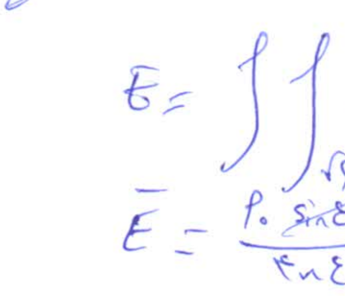
$\rho_v = \rho_0\cos\varphi$

$r = -r\hat{a}_r \rightarrow \bar{E} = \int_{\varphi=3\pi/2}^{2\pi}\int_{\theta=0}^{\pi/2}\int_{r=0}^{1}\frac{\rho_0\cos\varphi\, r^2\sin\theta\,dr\,d\theta\,d\varphi\,(-r\hat{a}_r)}{4\pi\varepsilon_0 r^3}$

$dv = r^2\sin\theta\,dr\,d\theta\,d\varphi$

$\bar{E} = \frac{\rho_0}{4\pi\varepsilon_0}\int_{3\pi/2}^{2\pi}\cos\varphi\,d\varphi\int_{\theta=0}^{\pi/2}\sin\theta\,d\theta\int_{r=0}^{1}dr\,(-\hat{a}_r) = \frac{\rho_0}{4\pi\varepsilon_0}(-\hat{a}_r)$

د)



$\rho_{s\theta} = \rho_0$

$ds_\theta = r\sin\theta\,dr\,d\varphi$

$\bar{r} = -r\hat{a}_r \qquad \theta = 60^\circ$

$E = \iint \frac{\rho_0\, r\sin\theta\,dr\,d\varphi\,(-r\hat{a}_r)}{4\pi\varepsilon_0 r^3} = \frac{\rho_0\sin\theta}{4\pi\varepsilon_0}\int_{r=0}^{\sqrt{2}}\frac{dr}{r}\int_0^{\pi/2}d\varphi\,(-\hat{a}_r)$

$\bar{E} = \frac{\rho_0\sin 60}{4\pi\varepsilon_0}\times(\ln\sqrt{2} - \ln 0)\times\frac{\pi}{2}(-\hat{a}_r)$

(الف) $V = \int\int \frac{\rho_0 \cos\varphi \, R\, dR\, d\varphi}{4\pi\varepsilon_0 R} = \frac{\rho_0}{4\pi\varepsilon_0}\int \cos\varphi\, d\varphi \int_1^2 dR = \frac{\rho_0}{4\pi\varepsilon_0}$ ②

ب) $V_1 = \int_{\varphi=0}^{\pi/2} \frac{\rho_0 \rho\, d\varphi}{4\pi\varepsilon_0 \rho} = \frac{\rho_0}{4\pi\varepsilon_0}\int_0^{\pi/2} d\varphi = \frac{\rho_0}{8\varepsilon_0}$

$V_2 = V_c = \int_{R=1}^{2} \frac{\rho_0\, dR}{4\pi\varepsilon_0 R} = \frac{\rho_0}{4\pi\varepsilon_0}\int \frac{dR}{R} = \frac{\rho_0 \ln 2}{4\pi\varepsilon_0}$

$V = \frac{\rho_0}{8\varepsilon_0} + \frac{\rho_0 \ln 2}{4\pi\varepsilon_0}$

ج) $V = \int_{3\pi/2}^{2\pi}\int_{0}^{\pi/2}\int_{r=0}^{1} \frac{\rho_0 \cos\varphi\, r^2 \sin\theta\, dr\, d\theta\, d\varphi}{4\pi\varepsilon_0 r} = \frac{\rho_0}{4\pi\varepsilon_0}\int_0^1 r\, dr \int_0^{\pi/2} \frac{d\theta}{\sin\theta} \int \cos\varphi\, d\varphi$

$V = \frac{\rho_0}{8\pi\varepsilon_0}$

د) $V = \int_{r=0}^{\sqrt{2}}\int_{\varphi=0}^{\pi/2} \frac{\rho_0 r \sin\theta\, dr\, d\varphi}{4\pi\varepsilon_0 r} = \frac{\rho_0 \sin\theta}{4\pi\varepsilon_0}\int_0^{\sqrt{2}} dr \int_0^{\pi/2} d\varphi$

$V = \frac{\rho_0}{4\pi\varepsilon_0} \, 2\, \frac{\pi}{2} = \frac{\rho_0}{8\varepsilon_0}$

④

$r > a \quad D(4\pi r^2) = Q \rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r$

$a < r < b$ داخل فلز $D = 0 \rightarrow E = 0$

$r < b \quad \oint D \cdot ds = Q \rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r$

$V = -\int E \cdot dl = -\int_\infty^a \frac{Q}{4\pi\varepsilon_0 r^2} dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{\infty}\right)$

پتانسیل داخل $= -\int_\infty^a E \cdot dl - \int_a^b E \cdot dl - \int_b^r E \cdot dl = \frac{Q}{4\pi\varepsilon_0 a} + \frac{Q}{4\pi\varepsilon_0 r} - \frac{Q}{4\pi\varepsilon_0 b}$