Laguerre polynomial approach for solving Lane–Emden type functional differential equations

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A R T I C L E   I N F O

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Collocation method
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A B S T R A C T

In this paper, a numerical method, which is called the Laguerre collocation method, for the approximate solution of Lane–Emden type functional differential equations in terms of Laguerre polynomials are derived. The method is based on the matrix relations of Laguerre polynomials and their derivatives, and reduces the solution of the Lane–Emden type functional differential equation to the solution of a matrix equation corresponding to system of algebraic equations with the unknown Laguerre coefficients. Also, some illustrative examples are included to demonstrate the validity and applicability of the proposed method.

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1. Introduction

Lane–Emden type equations model many phenomena in mathematical physics and astrophysics such as thermal explosions [1], stellar structure [2], the thermal behavior of a spherical cloud of gas, isothermal gas spheres, and thermionic currents [3,4] are derived. In this paper, by means of the matrix relations between the Laguerre polynomials and their derivatives, the numerical method given by Gürbüz et al. [5] is modified and developed for solving Lane–Emden type functional differential equation

\[ y''(\gamma x + \tau) + \frac{r}{x} y'((\beta x + \eta) + p(x)y(\alpha x + \mu)) = g(x), \quad 0 \leq x \leq b < \infty \]  

(1)

under the initial conditions

\[ y(0) = \lambda_0, \quad y''(0) = \lambda_1, \quad 0 \leq x \leq b < \infty \]  

(2)

and the solution is expressed in the truncated Laguerre series form

\[ y(x) = \sum_{n=0}^{N} a_n L_n(x), \quad 0 \leq x \leq b < \infty. \]  

(3)

Here, \( N \) is chosen any positive integer such that \( N \geq 2 \). \( a_n, n = 0, 1, 2, \ldots, N \) are unknown Laguerre coefficients, and \( L_n(x) \) are the Laguerre polynomials defined by

\[ L_n(x) = \sum_{r=0}^{n} \frac{(-1)^{n}}{r!} \binom{n}{r} x^r, \quad 0 \leq x \leq b < \infty. \]  

(4)

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The rest of this paper is organized as follows. We describe the formulation of Laguerre polynomials required for our subsequent development in Section 2. Second-order linear differential–difference equation with variable coefficients and fundamental relations are presented in Section 3. The new scheme are based on Laguerre collocation method and this method is described in Section 4. To support our findings, we present the result of numerical experiments in Section 5. Section 6 concludes this article with a brief summary. Finally, some references are introduced at the end.

2. Properties of Laguerre polynomials

A total orthonormal sequence in $L^2(-\infty, b]$ or $L^2[a, +\infty)$ can be obtained from such a sequence in $L^2[0, +\infty)$ by transformations $x = b - t$ and $x = t + a$, respectively. We consider $L^2[0, +\infty)$. Applying the Gram–Schmidt process to the sequence defined by

$$e^{-x/2}, \quad xe^{-x/2}, \quad x^2e^{-x/2}, \ldots,$$

we can obtain an orthonormal sequence $(s_n)$. It can be shown that $(s_n)$ is total in $L^2[0, +\infty)$ and is given by

$$s_n(x) = e^{-x/2}L_n, \quad n = 0, 1, 2, \ldots,$$

where the Laguerre polynomial of order $n$ is defined by

$$L_0(x) = 1, L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x}), \quad n = 1, 2, \ldots$$

The Laguerre polynomials $L_n(x)$ are solutions of the Laguerre differential equation [6]

$$xL_n''(x) + (1 - x)L_n'(x) + nL_n(x) = 0.$$ 

3. Fundamental relations

Let us consider Eq. (1) and find the matrix forms of the equation. First we can write Laguerre polynomials (4) in the matrix form

$$L^T(x) = HX^T \Rightarrow L(x) = X(x)H^T,$$

where $L(x) = [L_0(x) \ L_1(x) \ \ldots \ L_N(x)]$, $X(x) = [1 \ \ldots \ x^N]$ and

$$H = \begin{bmatrix}
-\frac{1}{0} & 0 & 0 & \cdots & 0 \\
\frac{-1}{0} & 1 & 0 & \cdots & 0 \\
\frac{-1}{0} & 2 & \frac{1}{1} & \cdots & 0 \\
\frac{-1}{0} & 3 & \frac{2}{2} & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{-1}{0} & N & \frac{N}{1} & \cdots & \frac{N}{N}
\end{bmatrix}$$

the solution $y(x)$ defined by a truncated Laguerre series (3)

$$y(x) = L(x)A,$$

where

$$A = [a_0 \ a_1 \ \ldots \ a_N]^T.$$ 

By using expression (5) and (6) the matrix relation is defined

$$y(x) = X(x)H^T A.$$ 

Also, relations between the matrix $X(x)$ and its derivatives $X'(x)$ and $X''(x)$ are

$$X'(x) = X(x)B^T \quad \text{and} \quad X''(x) = X(x)(B^T)^2,$$

where

$$B = \begin{bmatrix}
0 & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \cdots & 0 \\
0 & 2 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & N & 0
\end{bmatrix}.$$
By using the relations (7) and (8), we have the matrix relations
\[ y'(x) = X(x)B^T H^T A \]  
(9)
and
\[ y''(x) = X(x)(B^T)^2 H^T A. \]  
(10)
By substituting \( x \to ax + \mu \) in the relation (7)
\[ y(ax + \mu) = X(ax + \mu)H^T A. \]  
(11)
To obtain the matrix \( X(ax + \mu) \) in terms of the matrix \( X(x) \), we can use the following relation
\[ X(ax + \mu) = X(x)B_{(x,\mu)}, \]  
(12)
where for \( x \neq 0 \) and \( \mu \neq 0 \)
\[
B_{(x,\mu)} = \begin{bmatrix}
0 & (x)^0(\mu)^0 & 1 & (x)^0(\mu)^1 & 2 & (x)^0(\mu)^2 & \ldots & N & (x)^0(\mu)^N \\
0 & (x)^1(\mu)^0 & 2 & (x)^1(\mu)^1 & 2 & (x)^1(\mu)^2 & \ldots & N & (x)^1(\mu)^N-1 \\
0 & 0 & 2 & (x)^2(\mu)^0 & 2 & (x)^2(\mu)^1 & \ldots & N & (x)^2(\mu)^N-2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & 0 & \ldots & (x)^N
\end{bmatrix}
\]
for \( x \neq 0 \) and \( \mu = 0 \)
\[
B_{(x,0)} = \begin{bmatrix}
(x)^0 & 0 & 0 & \ldots & 0 \\
0 & (x)^1 & 0 & \ldots & 0 \\
0 & 0 & (x)^2 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & (x)^N
\end{bmatrix}
\]
from the relations between (11) and (12)
\[ y(ax + \mu) = X(x)B_{(x,\mu)}H^T A. \]  
(13)
Similar to relation (13), by substituting \( x \to bx + \eta \) in the relation (9)
\[ y'(bx + \eta) = X(bx + \eta)B^T H^T A \]
and from the relation (12) we have
\[ y'(bx + \eta) = X(x)B_{(b,\eta)}B^T H^T A. \]  
(14)
Consequently, by substituting \( x \to cx + \tau \) in the relation (10), the matrix relation is obtained
\[ y''(cx + \tau) = X(x)B_{(c,\tau)}(B^T)^2 H^T A. \]  
(15)

4. Method of solution

In this section, we construct the fundamental matrix equation corresponding to (15). For this purpose, we substitute the matrix relations given in the relations (13)–(15) into equation
\[ X(x)B_{(c,\tau)}(B^T)^2 H^T A + \frac{r}{x} X(x)B_{(b,\eta)}B^T H^T A + p(x)X(x)B_{(x,\mu)}H^T A = g(x). \]  
(16)
By using collocation points \( x_i \) in (16) defined by
\[ x_i = \frac{b}{N}i, \quad i = 0, 1, \ldots, N, \]  
(17)
where \( b \) is an integer which is smaller than infinity. Then, we get the system of matrix equations
\[ X(x_i)B_{(c,\tau)}(B^T)^2 H^T A + \frac{r}{x_i} X(x_i)B_{(b,\eta)}B^T H^T A + p(x_i)X(x_i)B_{(x,\mu)}H^T A = g(x_i). \]  
(18)
or briefly the fundamental matrix equation

\[
\left\{ XB_{\gamma;c} (B')^2 + RXB_{(\beta;\rho)} B' + PXB_{(\alpha;\mu)} \right\} H'A = G,
\]

where

\[
R = \begin{bmatrix} \frac{r}{s_0} & 0 & 0 & \ldots & 0 \\ 0 & \frac{r}{s_1} & 0 & \ldots & 0 \\ 0 & 0 & \frac{r}{s_2} & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \frac{r}{s_N} \end{bmatrix}, \quad P = \begin{bmatrix} p(x_0) & 0 & 0 & \ldots & 0 \\ 0 & p(x_1) & 0 & \ldots & 0 \\ 0 & 0 & p(x_2) & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & p(x_N) \end{bmatrix}, \quad G = \begin{bmatrix} g(x_0) \\ g(x_1) \\ g(x_2) \\ \vdots \\ g(x_N) \end{bmatrix}
\]

and

\[
X = \begin{bmatrix} X(x_0) \\ X(x_1) \\ X(x_2) \\ \vdots \\ X(x_N) \end{bmatrix} = \begin{bmatrix} 1 & x_0 & x_0^2 & \ldots & x_0^N \\ 1 & x_1 & x_1^2 & \ldots & x_1^N \\ 1 & x_2 & x_2^2 & \ldots & x_2^N \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_N & x_N^2 & \ldots & x_N^N \end{bmatrix}
\]

Hence, the fundamental matrix equation (19) corresponding to Eq. (1) can be written in the form

\[
WA = G|W, G|, W = [w_{ij}], \quad i, j = 0, 1, \ldots, N,
\]

where

\[
W = XB_{\gamma;c}(B')^2 + RXB_{(\beta;\rho)} B' + PXB_{(\alpha;\mu)} H'.
\]

Here, Eq. (14) corresponds to a system of \((N + 1)\) linear algebraic equations with unknown Laguerre coefficients. We have the conditions in matrix form [7]

\[
\sum_{k=0}^{1} a_{jk} y^{(k)}(0) + b_{jk} y^{(k)}(b) = \lambda_j, \quad j = 0, 1,
\]

where

\[
y^{(k)}(0) = X(0)(B')^k H'A \equiv \begin{bmatrix} u_{00} & u_{01} & \cdots & u_{0N} \end{bmatrix} = [\lambda_0],
\]

\[
y^{(k)}(b) = X(b)(B')^k H'A \equiv \begin{bmatrix} u_{10} & u_{11} & \cdots & u_{1N} \end{bmatrix} = [\lambda_1].
\]

On the other hand, the matrix form for conditions can be written as

\[
U_jA = [\lambda_j] \text{ or } [U_j; \lambda_j], \quad j = 0, 1,
\]

where

\[
U_j = \sum_{k=0}^{1} a_{jk} y^{(k)}(0) + b_{jk} y^{(k)}(b) \begin{bmatrix} \lambda_j \\ \lambda_{j+1} \end{bmatrix} \begin{bmatrix} B' \end{bmatrix}^k H'TA,
\]

\[
= \begin{bmatrix} w_{00} & w_{01} & \cdots & w_{0N} \\ w_{10} & w_{11} & \cdots & w_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ w_{N-2,0} & w_{N-2,1} & \cdots & w_{N-2,N} \\ u_{00} & u_{01} & \cdots & u_{0N} \\ u_{10} & u_{11} & \cdots & u_{1N} \end{bmatrix} \begin{bmatrix} g(x_0) \\ g(x_1) \\ \vdots \\ g(x_{N-2}) \\ \lambda_0 \\ \lambda_1 \end{bmatrix}, \quad j = 0, 1.
\]

To obtain the solution of Eq. (1) under conditions (2), by replacing the row matrices (23) by the last two rows of the matrix (20), we have the new augmented matrix,

\[
[W; \hat{G}] = \begin{bmatrix} w_{00} & w_{01} & \cdots & w_{0N} & g(x_0) \\ w_{10} & w_{11} & \cdots & w_{1N} & g(x_1) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ w_{N-2,0} & w_{N-2,1} & \cdots & w_{N-2,N} & g(x_{N-2}) \\ u_{00} & u_{01} & \cdots & u_{0N} & \lambda_0 \\ u_{10} & u_{11} & \cdots & u_{1N} & \lambda_1 \end{bmatrix}
\]

We solve this system with direct method given in the literature as LU factorization method and Faddeev–Leverrier formula. If

\[
\text{rank} W = \text{rank} [W; \hat{G}] = N + 1,
\]

then we can write [8]
Thus the matrix $A$ is uniquely determined. Also the Eq. (1) with conditions (2) has a unique solution. This solution is given by truncated Laguerre series. We can easily check the accuracy of the method. Since the truncated Laguerre series (3) is an approximate solution of Eq. (1), when the solution $y_n(x)$ and its derivatives are substituted in Eq. (1), the resulting equation must be satisfied approximately; that is, for $x = x_q \in [0, b]$, $q = 0, 1, 2, \ldots$

$$E(x_q) = \left| y''(x_q + \tau) + \frac{r}{x_q} y''(\beta x_q + \eta) + p(x_q) y(x + \eta) - g(x_q) \right| \approx 0 \quad (24)$$

and $E(x_q) \leq 10^{-k}$ ($k$ positive integer). If max $10^{-k} = 10-k$ ($k$ positive integer) is prescribed, then the truncation limit $N$ is increased until the difference $E(x_q)$ at each of the points becomes smaller than the prescribed $10^{-k}$. On the other hand, the error can be estimated by the function [9]

$$E_N(x) = y''(x + \tau) + \frac{r}{x} y''(\beta x + \eta) + p(x) y(x + \mu) - g(x) \quad (25)$$

If $E_N(x) \rightarrow 0$, when $N$ is sufficiently large enough, then the error decreases.

5. Illustrative examples

In this section, several numerical examples are given to illustrate the accuracy and effectiveness properties of the method and all of them were performed on the computer using a program written in Maple 12. The absolute errors in Tables are the values of $e_n(x) = |y(x) - y_n(x)|$ at selected points.

**Example 1.** Firstly, consider the Lane–Emden type differential–difference equation

$$y''(2x - 1) + \frac{2}{x} y'(3x) + xy(x - 1) = x^4 - 5x^3 + 7x^2 + 63x - 34, \quad 0 \leq x \leq 1 \quad (26)$$

with conditions

$$y(0) = 0, \quad y'(0) = 0.$$ 

Here is $p(x) = x, \ t = 2, \ g = 2, \ \tau = -1, \beta = 3, \eta = 0, \ x = 1, \mu = -1$, and $g(x) = x^4 - 5x^3 + 7x^2 + 63x - 34$. Now, let us seek the solution $y(x)$ as a truncated Laguerre series by putting $N = 3$ in Eq. (3)

$$y(x) = \sum_{n=0}^{3} a_n L_n(x).$$

For this purpose, the set of collocation points (17) is calculated for $N = 3$

$$\{ x_0 = 0, \ x_1 = 1/3, \ x_2 = 2/3, \ x_3 = 1 \}$$

and from Eq. (26), we obtain the fundamental matrix equation of the problem as

$$\{ XB(2, -1)B^T + RXB(3, 0)B^T + PXB(1, -1) \} H^T A = G.$$ 

where

$$B_{(2, -1)} = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 2 & -4 & 6 \\ 0 & 0 & 4 & -12 \\ 0 & 0 & 0 & 8 \end{bmatrix}, \quad B_{(3, 0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 27 \end{bmatrix}, \quad B_{(1, -1)} = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$R = \begin{bmatrix} 2/(0) & 0 & 0 & 0 \\ 0 & 2/((1/3)) & 0 & 0 \\ 0 & 0 & 2/((2/3)) & 0 \\ 0 & 0 & 0 & 2/(1) \end{bmatrix}, \quad P = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 2/3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix},$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & -2 & 1/2 & 0 \\ 1 & -3 & 3/2 & -1/6 \end{bmatrix}, \quad X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1/3 & 1/9 & 1/27 \\ 1 & 2/3 & 4/9 & 8/27 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad G = \begin{bmatrix} -34 \\ -1004/81 \\ -796/81 \\ 32 \end{bmatrix}.$$
Hence we compute the augmented matrix as
\[
\begin{bmatrix}
0 & -2 & 3 & 2 & \vdots & -34 \\
1/3 & (-2/1/3) + 5/9 & 50/27 - (2/(1/3)) & 1111/243 - 1/(1/3) & \vdots & -1004/81 \\
2/3 & (-2/(2/3)) + 8/9 & 58/27 - (1/(2/3)) & 1000/243 + 2/(2/3) & \vdots & -796/81 \\
1 & 0 & 0 & 0 & \vdots & -32
\end{bmatrix}
\]

From Eq. (26) the matrix forms of initial conditions are
\[
[u_0; \lambda_0] = [1 \quad 0 \quad 0 \quad 0 ; 0]
\]
and
\[
[u_1; \lambda_1] = [0 \quad -1 \quad -2 \quad -3 ; 0]
\]
Therefore we calculate the new augmented matrix based on the conditions as
\[
\begin{bmatrix}
0 & -2 & -3 & -2 & \vdots & -34 \\
1/3 & (-2/(1/3)) + 5/9 & 50/27 - (2/(1/3)) & 1111/243 - 1/(1/3) & \vdots & -1004/81 \\
1 & 0 & 0 & 0 & \vdots & 0 \\
1 & 0 & 0 & 0 & \vdots & 0
\end{bmatrix}
\]
Solving this system,
The Lane–Emden type differential–difference equation is computed by using the procedure in Section 4. Hence, linear algebraic system is gained. This system is approximately solved using the Maple 12. The results for \( N = 3, 4 \) are obtained by using the Laguerre collocation method discussed in Section 4. Finally, we substitute the elements of the Laguerre coefficients matrix \( A \) into Eq. (19). Thus, the approximate solution for \( N = 3 \) of Eq. (26) becomes

\[
y_2(x) = 2.10^{-19} - 1.99999999999999996x^2 + 0.999999999999999988x^3
\]

which is an approximate solution. The exact solution of (26) is \( y_2(x) = x^3 - 2x^3 \).

We display a plot of the exact and approximate solutions in Fig. 1 and error functions for various \( N \) is shown in Fig. 2. Table 1 shows solution of the problem for various \( N \).

**Example 2.** Lane–Emden equation is

\[
y''(x) + \frac{2}{x}y'(x) - 2(2x^2 + 3)y(x) = 0, \quad 0 \leq x \leq 1
\]  

(27)

with \( y(0) = 1, y'(0) = 0 \). Using the procedure in Section 4 and taking \( N = 8, 10, 12 \) the matrices in Eq. (27) are computed. The exact solution of (27) is \( y(x) = e^{x^2} \).

A plot of absolute difference exact and approximate solutions is displayed in Fig. 3, and error functions for various \( N \) is shown in Fig. 4. The solution of Lane–Emden equation is obtained for \( N = 8, 10, 12 \). For numerical results, see Table 2.

\[
A = [\frac{1}{500} \quad 0 \quad -499/250 \quad 1]^T
\]
Example 3. We apply the mentioned technique to solve the problem

\[ y''(3x - 1) + \frac{2}{x}y'(2x) + xy(x + 1) = x^4 + 3x^3 + 3x^2 + 44x - 6 \]  

with initial conditions

\[ y(0) = 1, \quad y'(0) = 0. \]

Table 2

<table>
<thead>
<tr>
<th>x</th>
<th>Exact solution</th>
<th>Present method</th>
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<tr>
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Fig. 4. Error function of Example 2 for various N.

Fig. 5. Numerical and exact solution of the Example 3 for N = 3, 9, 11.
where is the $0 \leq x \leq 1$ and this equation has the exact solution of $y(x) = x^3 + 1$. Using the same procedure for $N = 3, 9$ and 11 (28) are computed (see Fig. 5).

We display error functions of these methods in Fig. 6. We compare exact solutions with their error functions in Table 3.

### Table 3
Error analysis of Example 3 for the $x$ value.

<table>
<thead>
<tr>
<th>$x$</th>
<th>Exact solution</th>
<th>Present method</th>
<th>Present method</th>
<th>Present method</th>
</tr>
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</table>

6. Conclusion

In this study, we presented the numerical approach to find the solutions of a class of Lane–Emden type differential difference equations in terms of Laguerre polynomials of the first kind and the collocation points. The numerical approach was illustrated by accurately solving Lane–Emden equations. Illustrative examples are included to demonstrate the validity and applicability of the technique and performed on the computer using a program written in Maple 12. To get the best approximating solution of the equation, we take more forms from the Laguerre expansion of functions, that is, the truncation limit $N$ must be chosen large enough. In addition, an interesting feature of this method is to find the analytical solutions if the equation has an exact solution that is a polynomial functions [10]. Consequently, the obtained solutions for various particular cases demonstrate the validity and applicability of the method compare to the other existing methods [11–18]. We predict that the Laguerre expansion method will be a promising method for investigating analytic solutions to Lane–Emden equation.

References


