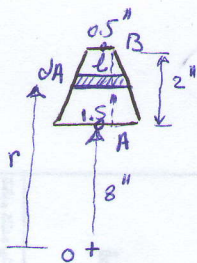


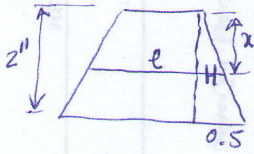
تکلیف درک هشتم مباحث مصالح I

سوال ۱۱



$$A = \frac{1}{2} (2) (0.5 + 1.5) = 2 \text{ (in}^2\text{)}$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{2}{\int \frac{dA}{r}}$$

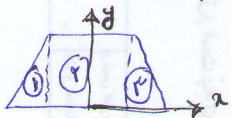


$$\frac{H}{0.5} = \frac{x}{2} \quad H = 0.25x = 0.25(10-r) \quad ; \quad d = 0.5 + 2H = 0.5 + 0.5(10-r) = 0.5(11-r)$$

فاصله از فرضی تا مرکز دوران

$$\Rightarrow \int \frac{dA}{r} = \int \frac{0.5(11-r)}{r} dr = 0.5 \int \left(\frac{11}{r} - 1 \right) dr = 0.5 \left(11 \ln r - r \right) \Big|_8^{10} = 0.227 \Rightarrow R = \frac{2}{0.227} = 8.811''$$

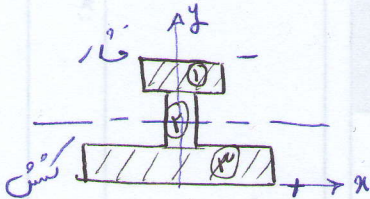
برای محاسبه $\bar{y}$ با استفاده از اصل مرکز ثقل را مشخص کنیم.



نقطه	A (in ²)	$\bar{y}$	A $\bar{y}$
①	0.5	0.667	0.334
②	1	1	1
③	0.5	0.667	0.334
	2		1.668

$$\Rightarrow \bar{y} = \frac{1.668}{2} = 0.834'' \Rightarrow \bar{r} = 8 + \bar{y} = 8.834''$$

$$\sigma = \frac{my}{A(\bar{r}-R)(R-y)} = \frac{600y}{2(8.834-8.811)(8.811-y)} = 13.043 \times 10^3 \frac{y}{8.811-y} \rightarrow \begin{cases} \sigma_A = 13.043 \frac{0.834}{7.977} = 1.364 \text{ ksi} \\ \sigma_B = 13.043 \frac{1.166}{7.645} = 1.989 \text{ ksi} \end{cases}$$



$$M = (6)(160) = 960 \text{ N.m}$$

سوال ۱۲

نقطه	A (m ²)	$\bar{y}$ (m)	A $\bar{y}$
①	7.5×10^{-4}	0.165	1.238×10^{-4}
②	1.5×10^{-3}	0.085	1.275×10^{-4}
③	1.5×10^{-3}	5×10^{-2}	0.075×10^{-4}
	3.75×10^{-3}		2.588×10^{-4}

$$\Rightarrow \bar{y} = \frac{2.588 \times 10^{-4}}{3.75 \times 10^{-3}} = 0.069 \text{ (m)} = 69.01 \text{ (mm)}$$

$$\Rightarrow \bar{r} = 0.25 + 0.069 = 0.319 \text{ (m)}$$

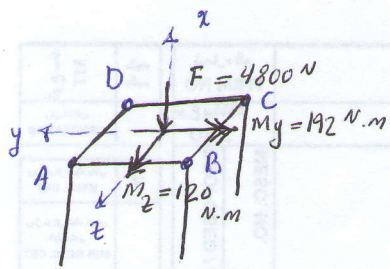
$$\int \frac{dA}{r} = \int_{0.25}^{0.42} \frac{dA}{r} + \int_{0.26}^{0.41} \frac{dA}{r} + \int_{0.41}^{0.42} \frac{dA}{r} = \int_{0.25}^{0.26} \frac{0.15 dr}{r} + \int_{0.26}^{0.41} \frac{0.01 dr}{r} + \int_{0.41}^{0.42} \frac{0.075 dr}{r} = 0.012 \text{ (m)}$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{3.75 \times 10^{-3}}{0.012} = 0.313 \text{ (m)}$$

$$\sigma = \frac{my}{A(\bar{r}-R)(R-y)} = \frac{960y}{(3.75 \times 10^{-3})(0.319-0.313)(0.313-y)} = 42.667 \frac{y}{(0.313-y)} \text{ MPa}$$

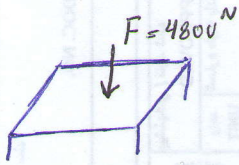
$$\begin{cases} \sigma_{max}^c = 42.667 \frac{0.101}{0.212} = 20.327 \text{ MPa} \\ \sigma_{max}^t = 42.667 \frac{0.069}{0.244} = 12.066 \text{ MPa} \end{cases}$$

سوال ۱۳ ابتدا با سیم بندوی P را به مرکز لچ تنفع انتقال بدیم:

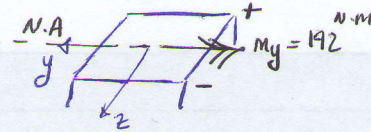


$$F = P = 4.8 \text{ kN}$$

$$\vec{M} = \vec{r} \times \vec{P} = (25\hat{j} + 40\hat{k}) \times (-4.8\hat{i}) = 120\hat{k} - 192\hat{j} \text{ (N.m)} \Rightarrow \begin{cases} M_y = -192 \text{ (N.m)} \\ M_z = 120 \text{ (N.m)} \end{cases}$$

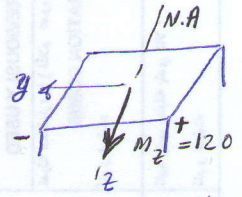


$$\sigma_1 = \frac{-F}{A} = \frac{-4800}{(0.12)(0.08)} = -0.5 \text{ MPa}$$



$$I_y = \frac{1}{12} (0.12)(0.08)^3 = 5.12 \times 10^{-6} \text{ (m}^4\text{)}$$

$$\sigma_y = -\frac{(192)z}{5.12 \times 10^{-6}} = -37.5z \text{ (MPa)}$$



$$I_z = \frac{1}{12} (0.08)(0.12)^3 = 1.152 \times 10^{-5} \text{ (m}^4\text{)}$$

$$\sigma_z = -\frac{(120)y}{1.152 \times 10^{-5}} = -10.417y \text{ (MPa)}$$

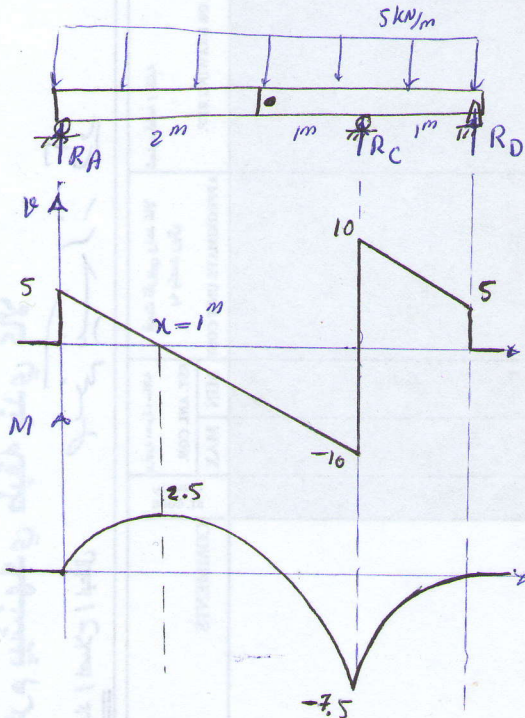
بیشترین فشاری در نقطه A است: $\sigma_c^{\max} = -0.5 - 37.5(0.04) - 10.417(0.06) = -2.625 \text{ (MPa)}$

بیشترین کشش در نقطه C است: $\sigma_t^{\max} = \sigma_c = -0.5 - 37.5(-0.04) - 10.417(-0.06) = 1.625 \text{ (MPa)}$

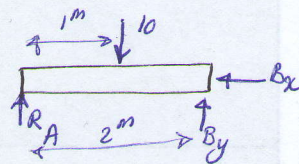
تفاوت بین: $\sigma = \sigma_1 + \sigma_y + \sigma_z = -0.5 - 37.5z - 10.417y$

را به خط نازقی:

$$\sigma = 0 \Rightarrow 0.5 + 37.5z + 10.417y = 0 \Rightarrow 75z + 20.834y = -1$$

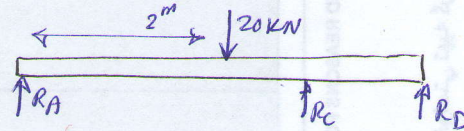


⇒



سوال ۱۴

$$\sum M_B = 0 \Rightarrow (1 \times 10) - (2 \times R_A) = 0 \Rightarrow R_A = 5 \text{ kN}$$



$$\sum M_D = 0 \Rightarrow (2 \times 20) - (4 \times R_A) - (1 \times R_C) = 0 \Rightarrow R_C = 20 \text{ kN}$$

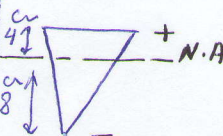
$$\sum F_y = 0 \Rightarrow R_A + R_C + R_D = 20 \Rightarrow R_D = -5 \text{ kN}$$

با توجه به نمودار تنش: $(C) m^{\max} = 7.5 \text{ (kN.m)}$

$$\frac{5}{10} = \frac{x}{3-x}$$

$$3-x = 2x \Rightarrow x = 1 \text{ m}$$

نقطه x:



$$I = \frac{1}{36} (0.08)(0.12)^3 = 3.84 \times 10^{-6} \text{ (m}^4\text{)}$$

$$\sigma_t^{\max} = \frac{(7.5 \times 10^3)(0.04)}{3.84 \times 10^{-6}} = 78.125 \text{ (MPa)}$$